MATH3543 Abstract Algebra II
Final Mock

December 2026

There are four questions in this paper. Questions 1 and 2 are worth 10 marks each. Questions 3 and 4 are worth 15 marks each. You must complete all questions for full marks.
Duration: 2 HOURS

Section 1: Solvability

1. (a) Define the terms *commutant* and *solubility* of a group G. (3)
   (b) Find the commutant of $D_{2\cdot 6}$, the group of symmetry preserving transformations of a regular hexagon. (7)

Section 2: Group Actions

2. (a) Define a *group action* of a group G on a set S. (3)
   (b) Let G be a group acting on a set S. Let $s \in S$ and consider the map (7)

$$\begin{aligned} f : &\{\text{left cosets of } \operatorname{Stab}(s)\} \to \operatorname{Orb}(s) \\ &: g \operatorname{Stab}(s) \mapsto gs. \end{aligned}$$

   Prove that f is a bijection.

Section 3: Sylow's Theorems

3. (a) State Sylow's theorems. (5)
   (b) Use Sylow's theorems, or any other method, to describe all cyclic subgroups of $D_{2\cdot 15}$. (10)

Section 4: Rings and Fields

4. (a) Determine whether (5)

$$\mathbb{Q}[x]/(x^2 - 4)$$

is a field. Justify your answer.

(b) Find the splitting field of the polynomial (10)

$$x^4 - 5x^2 + 6$$

over $\mathbb{Q}$, and determine its degree.